Approximate Stochastic Model of Geometry Randomness on Interconnect Parasitic in VLSI Circuit

Ying Yin[1,2], Xiaoyu Xu[1,2], Pengfei Lyu[1,2], Shuai Yan[1,2], and Zhuoxiang Ren[3]

[1]Institute of Microelectronics of Chinese Academy of Sciences, Beijing, China, xuxiaoyu@ime.ac.cn
[2]Beijing Key Laboratory of 3D & Nano IC Electronic Design Automation Technologies, Beijing, China
[3]Sorbonne Universités, UPMC Univ Paris 06, UR2, L2E, Paris, France

Fast and effective method is crucial to analyze the statistical characteristic of stochastic variation of interconnect parasitic in VLSI circuit. An approximate method of constructing an approximate stochastic model utilizing the first-order and second-order sensitivities to the process parameters is brought up, which can be used to handle the effect of geometry randomness on interconnect parasitic. This method is validated through an example of two conductor system. The serviceability and possible way of improvement are discussed.

***Index Terms*—Approximate stochastic model, geometry randomness, sensitivity analysis, interconnect parasitic**

I. Introduction

With the VLSI technology stepping into nano-scale era, process variations, including the material characteristics and the structural geometry, brought by the complicated manufacture procedure, such as the chemical mechanical polishing (CMP) and the lithography, have been major issues and hot spots of IC design and verification. In order to reduce the unforeseen impacts, the design should be insensitive to process variations and be checked by sensitivity verification [1], which requires suitable stochastic models. However, traditional methods, including the intrusive methods (*e.g.*, stochastic spectral finite element method [2]) and non-intrusive methods (*e.g.*, stochastic collocation method [3]) to a certain extent, are time-consuming. Considering that the sensitivity analysis is usually used to approximate the exact solution, it is utilized in this paper to evaluate the probability estimations of object stochastic variables (*e.g.*, interconnect capacitance) induced by process variations (*e.g.*, width fluctuation of a conductor).

II. Method of Sensitivity and Approximate Stochastic Model

A. Capacitance extraction

Capacitance extraction in IC is a typical electrostatic problem, which leads to a linear algebraic system Mv=b by the finite element method (FEM) where the material matrix M can be decomposed to

$$\mathrm{M}=\begin{bmatrix}\mathrm{M}_e & \mathrm{N}\\ \mathrm{N}^{\mathrm{T}} & \mathrm{L}\end{bmatrix},$$

where $\mathrm{M_e}$, N, and L are, respectively, the square, rectangular, and square matrices linking the unknown nodes, the unknown and boundary nodes, the boundary nodes [1]. Then the capacitances between conductors can be obtained easily after solving

$$\mathrm{M}_e v_e = -\mathrm{N} v_0 \,. \tag{1}$$

B. Sensitivity computation

The elements of the matrices M, $\mathrm{M_e}$, N and L are functions of process parameters p. Taking the derivative of Eq.(1), with respect to design parameters (e.g., p_r, and p_s), we obtain the first and the second order sensitivities, which read [1]

$$\mathrm{M}_e\frac{\partial v_e}{\partial p_r}=-\frac{\partial \mathrm{M}_e}{\partial p_r}v_e-\frac{\partial \mathrm{N}}{\partial p_r}v_0\,, \tag{2.1}$$

and

$$\mathrm{M}_e\frac{\partial^2 v_e}{\partial p_r\partial p_s}=-\frac{\partial \mathrm{M}_e}{\partial p_r}\frac{\partial v_e}{\partial p_s}-\frac{\partial^2 \mathrm{M}_e}{\partial p_r\partial p_s}v_e-\frac{\partial \mathrm{M}_e}{\partial p_s}\frac{\partial v_e}{\partial p_r}-\frac{\partial^2 \mathrm{N}}{\partial p_r\partial p_s}v_0\,. \tag{2.2}$$

C. Approximate Stochastic Model

Knowing the potential v_0 and working out the derivatives of potential to geometry parameters $\partial v/\partial p_r$, and $\partial^2 v/\partial p_r \partial p_s$, We can then depict the capacitance using Taylor series as Eq.(3) where $\mathrm{C_0}$ is the capacitance at the design node (mean node), $\partial C/\partial p_r$ is the sensitivity of capacitance C with respect to the process parameters p_r and *etc*.

$$C \approx C_0+\sum_r\frac{\partial C}{\partial p_r}\Delta p_r+\frac{1}{2}\sum_r\sum_s\frac{\partial^2 C}{\partial p_r\partial p_s}\Delta p_r\Delta p_s\,, \tag{3}$$

III. Numerical Experimentation and Results

Once $v(p)$, $\partial v(p)/\partial p_r$ and $\partial^2 v(p)/\partial p_r^2$ are acquired by FEM and sensitivity method, the 2nd order approximated model of object parameters, such as capacitance and electrical potential distribution, is formed. Based on this model instead of direct random sampling method, analysis of probability distribution of the object parameters can be achieved much more easily and concisely, which means moment estimators and probability distribution functions (*pdf*) of the object parameters on various orders can be obtained effectively without prominent loss in accuracy.

The *self* capacitance $\mathrm{C_{11}}$ in the multi-conductor electrostatic system as shown in Fig. 1 is investigated. The mesh involves 18,544 triangular elements and 9443 nodes. In this system the design parameter is the width W of conductor 1, which is supposed to be a random variable. Using Eqs. (1)-(3), we have

$$\begin{aligned}C_{11} &\approx C_0+c_1\Delta \mathrm{W}_1+c_2\Delta \mathrm{W}_1^{\,2}\\ &=0.3786+0.1527\Delta \mathrm{W}+0.05997\Delta \mathrm{W}^2\end{aligned}\,, \tag{4}$$

where $\mathrm{C_{11}}$ is the self-capacitance of conductor 1, $\mathrm{C_0}$ is mean of $\mathrm{C_{11}}$ when there is no variation of width, and c_1 and c_2 are respectively the first order sensitivity and second order sensitivity.

By Eq.(4), the probability parameters, such as mathematical expectations and variances, can be directly obtained as $E(C_{11}) \approx 0.3836$, $D(C_{11}) \approx 0.001963$ if the variation of W obey the uniform distribution U[-0.5, 0.5]. This method is simple and fast, it doesn't need sampling but we can't see the distribution of capacitance.

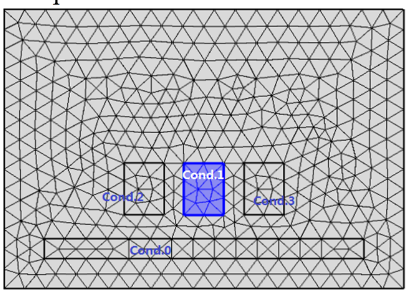

Fig 1. 4-conductors system, where 1 volt voltage is applied on conductor 1 and 0 volt voltage on others. C_{11} denotes the self-capacitance of conductor 1.

Direct Monte Carlo method (DMC) is another way to get the corresponding results and statistical characteristics. The DMC inquires capacitances by FEM Solver over numerous samples of the variations of width which is very time consuming. Here we use the DMC to validate our 2nd order sensitivity approach. With the DMC, we can obtain the mean and variation of the solutions corresponding with 100,000 samples, *i.e.*, $E(C_{11}) \approx 0.3786$ and $D(C_{11}) \approx 0.0002102$. The distributions of the width and the self-capacitance by the sensitivity model and by DMC are shown in Fig.2. The relative error between the sensitivity method and the DMC is 0.50%.

We can also adopt the second-order polynomial fitting method to obtain the second order model from the samples, which reads as

$$C_{11} \approx 0.3786 + 0.1581\Delta W + 0.05988\Delta W^2, \qquad (5)$$

and $E(C_{11}) \approx 0.3786$, $D(C_{11}) \approx 0.0002101$. The Eq.(5) and results are very close to those of the sensitivity model denoted by Eq. (4).

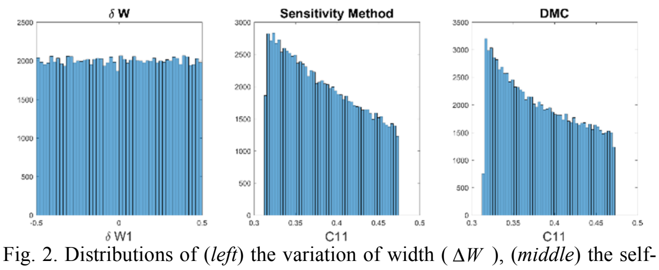

Fig. 2. Distributions of (*left*) the variation of width (ΔW), (*middle*) the self-capacitance C_{11} based on second-order sensitivity method, and (*right*) C_{11} based on direct Monte Carlo method. ΔW meets the uniform distribution, $\Delta W \sim U(-0.5, 0.5)$.

Considering the case that ΔW meets normal distribution $N(0, (1/3)^2)$, the expectations and variances of the sensitivity method and the DMC are, respectively, $E(C_{11}) \approx 0.3836$, and $E(C_{11}) \approx 0.3794$, $D(C_{11}) \approx 0.001963$, $D(C_{11}) \approx 0.0002970$. The distributions of the self-capacitance are compared in Fig. 3. The relative error between the sensitivity method and the DMC is 1.11%.

The results of the self-capacitance can be expressed as Eq.(6) via linear fit,

$$C_{11} \approx 0.3786 + 0.1547\Delta W + 0.05780\Delta W^2, \qquad (6)$$

which is also very close to the sensitivity model.

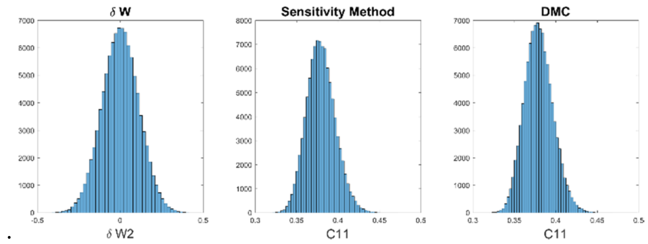

Fig. 3. Distributions of (*left*) the variation of width (ΔW), (*middle*) the self-capacitance C_{11} based on second-order sensitivity method, and (*right*) C_{11} based on direct Monte Carlo method. ΔW meets the normal distribution $\Delta W \sim N\left(0, (1/3)^2\right)$.

IV. Discussion and Conclusions

Computational analysis shows that if the variation is significant (more than 5%), the proposed method can be only used qualitatively as an approximate description, but we can construct several approximate stochastic models on different design nodes, and use them to compute the distribution piecewisely.

On the other side, we can compute the sensitivities using stochastic method such as Sobol methods and polynomial chaos [4][5]. In the full paper, the implementation and performance of such methods and the stochastic FEM are to be discussed.

Moreover, the relationship between different dimensions is usually ignored for calculation convenience, but it is not the fact. The correlation should be considered in both equation derivation processing and random sampling for multivariable system.

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